Zonal Vortex Method and Impulsively Started Flow Around a Circular Cylinder

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Abstract

IN this paper, a zonal vortex method and its application to simulating the impulsively started flow of a circular cylinder are presented. This method treats the attached viscous flow region and the separated flow region separately. The attached flow region is computed through solving boundary-layer equations by the finite-difference method. Only the separated flow is computed through solving Navier-Stokes equations by the vortex method. Since the separated flow region has a length scale of $\mathcal{O}(1)$, in the vortex method used for this region, the number of new vortices introduced at the surface of the body per time step is relatively insensitive to the Reynolds number of the flow. For simulation of high-Reynolds-number flows with massive separation, the total number of vortices in the flowfield, hence the computer storage and computer time, is greatly reduced.

Contents

The impulsively started flow around a circular cylinder is complex and all of the phenomena of fluid mechanics are presented. Because of its fundamental importance, this flow has become a typical problem in the study of separated flows. Bouard and Coutanceau[1] have done careful experimental studies on this problem and provided valuable data for comparison with numerical solutions. Smith and Stansby[2] have recently calculated this flow using a vortex method. In this method, the vorticity equation is solved using random walk for diffusion and the vortex-in-cell method for convection in a time-stepping procedure. Vortices are introduced at the surface at each time step to satisfy the no-slip condition. Their calculations revealed detailed flow features of the wake and were in good agreement with experimental observations in Ref. 1. Their numerical experiment showed that accurate simulation requires the introduction of a sufficiently large number of vortices at the surface per time step. This number increases as the Reynolds number increases. Therefore, for simulation of a high-Reynolds-number flow by the vortex method, the total number of vortices in the flow may become very large as time increases, requiring large computer storage and computer time.

In this paper, the vortex method is improved by a zonal approach. For a high-Reynolds-number flow with massive separation, the vorticity in the upstream of the separation point is confined in a layer near the surface of the body. The length scale of the thickness of this layer is $\mathcal{O}(Re^{-1/2})$. The length scale of the separated flow region is $\mathcal{O}(1)$. It is difficult to accommodate these two scales when solving the Navier-Stokes equations in the whole flow region. The approach of the present method is to treat the flow regions of different length scales separately. In the separated flow region, the Navier-Stokes equations are solved by the vortex method. Since the length scale of this flow region is $\mathcal{O}(1)$, the number of vortices introduced into the flow at the surface of the body per time step may become relatively insensitive to the flow Reynolds number. In the attached flow region, the boundary-layer equation is solved by the finite-difference method. By the above method, the total number of vortices in the flow, hence the computer storage and computer time, can be greatly reduced for high-Reynolds-number flow simulations. The method is applied to simulating the impulsively started flow of a circular cylinder.

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Description of the Method

The Navier-Stokes equations in vorticity/stream-function form may be written as

$$\frac{\partial \omega}{\partial t} + V \cdot \nabla \omega = \frac{2}{Re} \nabla^2 \omega \tag{1}$$

$$\nabla^2 \psi = -\omega \tag{2}$$

where the variables have been nondimensionalized with respect to freestream quantities and the radius a of the cylinder. The Reynolds number is defined as $Re = 2aV_\infty/\nu$. The boundary conditions required are the no-slip condition at the solid surface and the potential flow at outer boundary. The flowfield is divided into the attached and separated flow regions and the demarcation boundary is set somewhere at the upstream of the separation point. In the separated flow region, Eq. (1) is split into two parts and solved by the methods described in Ref. 2. In the attached viscous flow region, Eq. (1) is simplified to the boundary-layer equation. This equation is solved by Cebeci's finite-difference method.[3]

The velocity of the flow is obtained from solving Eq. (2) in a radially expanding polar mesh. The vorticity on each node of the mesh must be given. In the separated flow region, the circulation carried by the vortices is distributed over the nodal

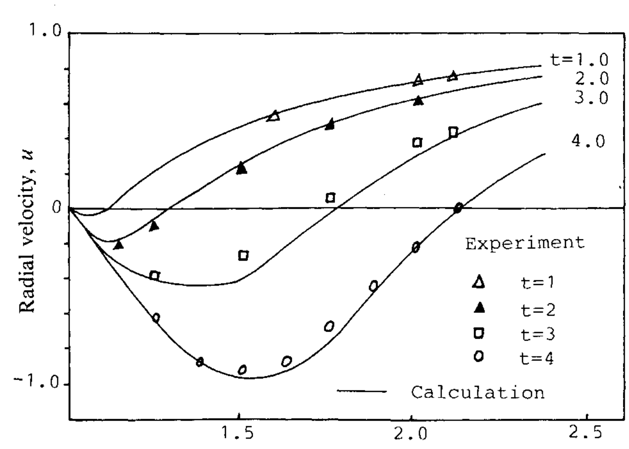

Fig. 1 Velocity profiles ($Re = 3000$).

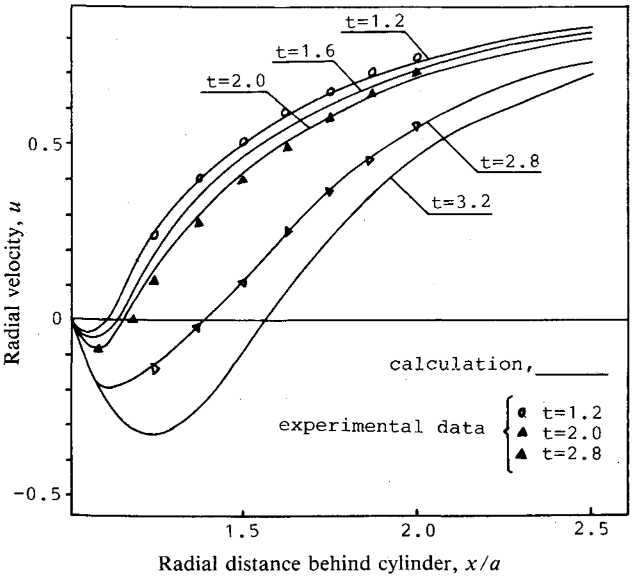

Fig. 2 Velocity profiles (Re = 9500).

points according to the vortex-in-cell method.[2] In the attached flow region, the vorticity is directly obtained from the solution of the boundary-layer equation.

In the separated flow region, the no-slip condition is satisfied by introducing vortices at the surface of the cylinder at each time step.[2] From Eq. (2), the vorticity at each surface nodal point can be calculated from the stream function. This vorticity is then converted into a circulation that is shared between the newly created vortices.[2] The numerical experiment of Ref. 2 showed that the number of new vortices created at each surface node n_v had to be increased as the flow Reynolds number increased. In the present approach, the vortex method is only used in the separated flow region that has a length scale of $\mathcal{O}(1)$. Therefore, it is hoped that n_v would be relatively insensitive to the flow Reynolds number. The vorticity on each nodal point on the demarcation boundary is known from the boundary-layer calculation, and it is treated in the same manner as the vorticities at the surface nodal points.

Results

Two cases of Reynolds number, $Re = 3000$ and 9500, are considered. For these Reynolds numbers, there are experimental data[1] and calculated results by the vortex method[2] for comparison. Following the results of the numerical experiments in Ref. 2, Δt is taken as 0.02 and a 65×65 mesh is used for the solution of the Poisson equation. The demarcation boundary is set at 90 deg from the rearward stagnation point.

Figure 1 gives profiles of the radial velocity on the symmetry axis behind the cylinder up to $t = 4.0$ for the case of $Re = 3000$. The results calculated by the zonal vortex method is in good agreement with the experimental data. In the calculation n_v is 3. In Ref. 2, using the vortex method for the whole flow region, for good results up to $t = 4.0$, $n_v = 5$ was required. In our calculations, since the vortex method is used only in the separated flow region and n_v can be smaller, the number of vortices in the flow is greatly reduced. At $t = 4.0$ in the preceding calculation, there are about 27,130 vortices in the flow. In the calculation of Ref. 2, this number is about 65,100, nearly 2.5 times as large as ours. Figure 2 gives the results for the case of $Re = 9500$. For this calculation, $n_v = 4$ was used. In the calculation of Ref. 2, $n_v = 6$ was required for the calculated results to be reliable at $t = 2$ to 2.8.

Acknowledgments

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